# Dynamical mass shift for a partially reflecting moving mirror

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**Abstract**

We consider the vacuum fluctuations contribution to the mass of a mirror in an exactly soluble partially reflecting moving mirror model. Partial reflectivity is accounted for by a repulsive delta-type potential localized along the mirror's trajectory. The mirror's mass is explicitly obtained as an integral functional of the mirror's past trajectory.

## 1 Introduction

Some years ago in a series of papers [1]-[5] Jaekel and Reynauds discussed quantum field theory implications to the inertial properties of partially reflecting mirrors. They evidenced that vacuum fluctuations lead to a shift in the mirror's mass, which is not happening for perfectly reflecting mirrors. The phenomenon can be intuitively understood by recalling a well-known situation in non-relativistic wave mechanics: scattering on barrier potentials is generally accompanied by a time delay, implying that for a certain time the energy of the scattered object can be considered to contribute to the proper energy of the scatterer. The mirror (taken here as a classical object) in the vacuum of the quantized field can be viewed as scattering the virtual zero-point oscillations. For perfect reflectors there is no time delay and thus no mass shift.

A particularly simple case evidencing the apparition of the mass shift is that of a pointlike mirror in 1+1 Minkowski space with the mirror-field interaction strictly localized at the mirror's position [4, 5]. The model was discussed in the cited papers basically at the classical level. Our intention here is to analyze the quantized version. We shall base our investigation on the results obtained in Ref. [6] where quantization of the massless scalar field corresponding to this model was achieved. The treatment therein has the convenient features (i) of being non-perturbative, and (ii) of relying on the explicitly constructed space-time dependent Heisenberg field operator. These make possible the *exact* evaluation of local quantities, such as the radiated energy-momentum density or correlation functions, at *arbitrary* times (by contrast, in Refs. [1]-[5] an in-out formalism[1] was used, and calculations were mainly performed in the small mirror displacements approximation).

Our basic result is the non-perturbative trajectory dependence of the mass shift. We take the view that the free motion quantity is ultimately unobservable, similar to the electromagnetic mass shift for the electron in Q.E.D., say. Hence, by definition uniform trajectories will be characterized by vanishing mass shifts. Practically, the model implies that the mass shift is given by the renormalized two point function evaluated at coincidence points at the mirror's instantaneous position. One naturally interprets it as originating in the distortion of the vacuum fluctuations produced by the mirror's (non-uniform) motion. For perfectly reflecting mirrors, absence of the mass shift can be equivalently seen as due to the "freezing" of the vacuum fluctuations along the trajectory, as the field operator itself vanishes at these points [11, 12]. A most notable feature is that the mass shift appears as a history dependent quantity. This is directly related to the same characteristic displayed by the quantum flux radiated by the mirror [6].

Another issue we discuss is the connection with the mirror dynamical equation, taking into account the quantum field backreaction. We point out that the mass shift appears from the lack of orthogonality between the mirror two-velocity and the backreaction force. Perfect mirrors prove to have this relation assured [11].

The paper is organized as follows. In Sec. 2 we briefly describe the mirror model. In Sec. 3 we relate the mass shift to the mirror dynamics. Considerations in this section are rather general, as they do not refer to a

[1] See Refs. [7]-[9].

particular mirror model. In Sec. 4 we obtain the trajectory dependence and evidence some of its general properties. Section 5 contains discussions concerning the positivity of the mass shift, which is left as an open problem.

# 2 The mirror model

Let $z^\mu$ ($\mu = 0, 1$) represent the coordinates in the $1+1$ Minkowski space[2] and $z^\mu(\tau)$ denote the mirror trajectory with $\tau$ the proper time

$$d\tau = \sqrt{dz^\mu dz_\mu}. \tag{1}$$

The mirror-field system is described by the action

$$S[\phi, z] = \frac{1}{2}\int d^2 z \partial_\mu \phi(z) \partial^\mu \phi(z) - m \int d\tau - \frac{a}{2}\int d\tau \phi^2(z(\tau)), \tag{2}$$

where $m$ is the mirror's mass and $a$ is a positive constant characterizing the mirror-field interaction. The field equation following from (2) is

$$(\Box + V(z))\phi(z) = 0, \tag{3}$$

with

$$V(z^0, z^1) = \frac{a}{\dot{z}^0(\tau)} \delta(z^1 - z^1(\tau_z)), \tag{4}$$

where $\delta$ denotes the Dirac distribution and $\tau_z$ is implicitely determined from equation $z^0(\tau_z) = z^0$. The overdot represents differentiation with respect to proper time. Eq. (4) is equivalent to a repulsive barrier potential localized on the trajectory with $a$ correponding to the barrier strength. One obtains thus a semitransparent moving mirror model with semitransparency controlled by parameter $a$.

Quantization of $\phi(z)$ respecting eq. (3) for $z^\mu(\tau)$ arbitrary was performed in Ref. [6]. We refer to this paper for the expressions of the in-vacuum two point function and the renormalized energy-momentum tensor.

[2] The metric tensor is $g_{00} = -g_{11} = 1$. Natural units $\hbar = c = 1$ are used throughout the paper.

Now, translational invariance of $S$ assures conservation of a total mirror-field energy-momentum

$$P_\mu = \int_{-\infty}^{+\infty} dz^1\, T_{\mu 0}(z) + [\, m + \frac{a}{2}\phi^2(z(\tau))\,]\, \dot{z}_\mu(\tau), \tag{5}$$

where

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\partial^\alpha\phi\partial_\alpha\phi, \tag{6}$$

and $\tau$ is arbitrary. Integration is taken along the spatial hypersurface $z^0 = z^0(\tau)$. One sees that the interaction term is equivalent to a $\phi$ dependent contribution to the mirror's mass

$$\mu = \frac{a}{2}\phi^2. \tag{7}$$

With $\phi$ the quantum field, eq. (7) defines the mass shift operator, as already discussed in Refs. [4, 5].

We shall assume in the following that the quantum field is in the vacuum state at infinite past. This ammounts to replace all operatorial quantities with their corresponding renormalized in-vacuum expectation values. We shall understand this tacitly in what follows, without a notational change. Our interest lies in the trajectory dependence of $\mu(\tau)$. Before going into details, we consider next the connection with the dynamical equation.

# 3 Dynamical equation and mass shift

Consider for the moment the free field contribution in eq. (5). We rewrite it as

$$P_\mu^{field}(\tau) = \int_{-\infty}^{z^1_-(\tau)} dz^1\, T^L_{\mu 0}(z) + \int_{z^1_+(\tau)}^{+\infty} dz^1\, T^R_{\mu 0}(z), \tag{8}$$

where

$$z^1_\pm(\tau) = z^1(\tau) \pm \epsilon, \tag{9}$$

with $\epsilon > 0$, $\epsilon \to 0$. Superscripts $L$, $R$ refer to the left and right regions of the Minkowski plane, as naturally determined by the trajectory. We excluded from integration an infinitesimal vecinity around $z^1(\tau)$, with no relevance for the total quantity. Note that the barrier potential implies that for points strictly on the trajectory $\partial_\mu \phi$ are discontinous, while $T_{\mu\nu}$ has a non-vanishing divergence.

The $\tau$-derivative of eq. (8) is minus the two-force acting on the mirror due to the field backreaction. In null coordinates $z^\pm = z^0 \pm z^1$ one has

$$\dot{P}_+^{field}(\tau) = T^L_{++}(z(\tau))\, \dot{z}^+(\tau), \tag{10}$$

$$\dot{P}_-^{field}(\tau) = T^R_{--}(z(\tau))\, \dot{z}^-(\tau), \tag{11}$$

where $z(\tau)$ in $T^L_{++}$ is understood as

$$(z^0(\tau), z^1_-(\tau)), \tag{12}$$

and analogously with $z^1_- \rightarrow z^1_+$ for $T^R_{--}$. To obtain relations above one uses first the divergenceless of $T_{\mu\nu}$ *inside* the $L$, $R$ regions to reduce the $z_1$-integrations to pure boundary terms at $z(\tau)$. One further takes into account that some of the $T_{\mu\nu}$ components vanish. By eq. (6) one has

$$T^{R,L}_{+-} = T^{R,L}_{-+} = 0. \tag{13}$$

Explicit calculations also show

$$T^R_{++} = T^L_{--} = 0. \tag{14}$$

(This states that there is no incoming flux from past null infinity, a direct consequence of choosing the field in the vacuum state at infinite past.) Using now total energy-momentum conservation one obtains

$$\frac{d}{d\tau}[m(\tau)\dot{z}_+(\tau)] + T^L_{++}(z(\tau))\, \dot{z}^+(\tau) = 0 \tag{15}$$

$$\frac{d}{d\tau}[m(\tau)\dot{z}_-(\tau)] + T^R_{--}(z(\tau))\, \dot{z}^-(\tau) = 0, \tag{16}$$

were we introduced the total mass

$$m(\tau) = m + \mu(\tau). \tag{17}$$

One can further eliminate $\ddot{z}^\pm$ from eqs. (15), (16) by using the othogonality relation

$$\ddot{z}_+ \dot{z}^+ + \ddot{z}_- \dot{z}^- = 0. \tag{18}$$

One finds

$$\dot{\mu}(\tau) + T^L_{++}(z(\tau))\,\dot{z}^+(\tau)^2 + T^R_{--}(z(\tau))\,\dot{z}^-(\tau)^2 = 0. \tag{19}$$

Hence, $\mu$ variations compensate for the non-orthogonality between the mirror velocity and the backreaction force.

Quantities $T^R_{--}$, $T^L_{++}$ were explicitly obtained in Ref. [6] as integral functionals of the mirror's past trajectory. When evaluated at $z(\tau)$ as above, trajectory dependence extends up to the proper time $\tau$, in agreement to causality. Eqs. (15)-(17) and (19) provide an integro-differential system determining the mirror dynamics (in the absence external forces). For our discussion, it is relevant eq. (19). It defines the mass shift in terms of the past mirror's trajectory, independently of the dynamical problem. We point out that eq. (19) could have been equivalently obtained from only manipulating the field equation (3) and using eq. (14). Considerations above were merely intended to make clear the dynamical origin of $\mu$.

# 4 Trajectory dependence of $\mu(\tau)$

Eq. (7) defines the mass shift in terms of the renormalized two point function evaluated at identical points $z(\tau)$. This yields[3]

$$\mu = \mu_1 + \mu_2, \tag{20}$$

where

$$\begin{aligned}
\mu_1(\tau) = \frac{a}{8\pi}\int_{-\infty}^{\tau} d\tau_1 \{ & \ln((z^-(\tau_1) - z^-(\tau))(z^+(\tau_1) - z^+(\tau)) \\
& \times \exp a(\tau_1 - \tau)/2 \,\} \\
+\frac{a}{8\pi}\int_{-\infty}^{\tau} d\tau_1 \{ & \ln((z^-(\tau) - z^-(\tau_1))(z^+(\tau) - z^+(\tau_1)) \\
& \times \exp a(\tau_1 - \tau)/2 \,\},
\end{aligned} \tag{21}$$

[3]See Ref. [6], eqs. (36)-(39). Please note that quantities therein are written for $a \to 2a$. $i\epsilon$ prescription is unnecessary here.

$$\mu_2(\tau) = -\frac{a^2}{16\pi}\int_{-\infty}^{\tau}\int_{-\infty}^{\tau} d\tau_1 d\tau_2 \{\, \ln((z^+(\tau_1) - z^+(\tau_2))$$

$$\times (z^-(\tau_1) - z^-(\tau_2)) \exp\, a((\tau_1 + \tau_2)/2 - \tau)\,\}. \tag{22}$$

Past history dependence is manifest. On the other side, the $\tau$-derivative of $\mu$ is determined by eq. (19) in terms of the renormalized energy-momentum tensor. One finds[4]

$$\dot\mu(\tau) = -\frac{a}{8\pi}\int_{-\infty}^{\tau}\int_{-\infty}^{\tau} d\tau_1 d\tau_2 \partial_{\tau_1}\partial_{\tau_2}\left(\frac{\dot z^+(\tau_1) - \dot z^+(\tau_2)}{z^+(\tau_1) - z^+(\tau_2)} + \frac{\dot z^-(\tau_1) - \dot z^-(\tau_2)}{z^-(\tau_1) - z^-(\tau_2)}\right)$$
$$\times \exp\, a((\tau_1 + \tau_2)/2 - \tau). \tag{23}$$

A straightforward calculation shows that eq. (23) results indeed from eqs. (20)-(21). We want to stress out that this does not trivially follows from eqs. (6), (7), (19), seen as defining an operatorial identity for $\phi$. Quantities above result from a renormalization procedure, implying the potentially dangerous infinite subtraction.

We evidence next some general properties of $\mu(\tau)$ following from eqs. (20)-(23).

One sees from eq. (23) that $\mu$ is constant along uniform trajectories. Explicit calculations using eqs. (20)-(22) yield the velocity-independent quantity

$$\mu_0 = \frac{a}{4\pi}\left(-\ln\frac{a}{2} + \int_0^{\infty} dx \ln x\, e^{-x}\right). \tag{24}$$

As mentioned in Sec. 1, we regard $\mu_0$ as an inobservable contribution to the physical mirror mass. Correspondingly, we redefine the mass shift as

$$\mu(\tau) \rightarrow \mu(\tau) - \mu_0. \tag{25}$$

Note that by eqs. (21), (22) $\mu$ contains the logarithm of a dimensionful quantity. This is remediated by subtraction (25). We recall that in quantizing $\phi$ the trajectories were supposed uniform in the infinite past. This is physically

[4] *ibidem*, eqs. (42)-(45).

not a serious restriction and can be always assumed in a realistic situation. Combined with eq. (25), it is clear that this requests

$$\lim_{\tau\to-\infty}\mu(\tau)=0, \tag{26}$$

which serves as the initial condition for eq. (23).

The exponentials in $\mu_1$, $\mu_2$ imply that $\mu(\tau)$ is essentially determined by the motion before $\tau$ in an interval of order $\sim a^{-1}$. This has an immediate consequence. Consider a trajectory with the velocity remaining constant after a fixed proper time $\tau_0$. One has then that for $\tau-\tau_0\gg a^{-1}$ the mass shift practically vanishes, as in the integrals (21), (22) the trajectory can be approximated with an uniform one.

The perfect reflectivity limit is obtained[5] by making $a\to\infty$. It's not hard to see that eq. (23) implies $\lim_{a\to\infty}\dot\mu(\tau)=0$. Taking into account condition (26) one concludes that the mass shift is absent for perfect mirrors, as pointed out in Sec. 1.

It is relevant to consider the case of slowly varying motions on a proper time scale of order $a^{-1}$, i.e.

$$|\alpha|\ll a^{-1},\quad \left|\frac{d^{n+1}\alpha}{d\tau^{n+1}}\Big/\frac{d^n\alpha}{d\tau^n}\right|\ll a^{-1},\quad n=0,1,2,\ldots, \tag{27}$$

where $\alpha$ denotes the mirror's proper acceleration. This is equivalent to "large" values for $a$, corresponding to a near perfect mirror behaviour. Then the paranthesis in eq. (23) is also a slowly varying function on intervals $\Delta\tau_{1,2}\sim a^{-1}$. Assuming the trajectory infinitely differentiable, one may Taylor expand it around $\tau_1=\tau_2=\tau$ and perform a term by term integration. The result is a $a^{-n}$ ($n\geq 1$) expansion with the coefficients entirely determined by the acceleration and its derivatives evaluated at $\tau$. We give below the $O(a^{-3})$ contribution

$$\dot\mu(\tau)=\frac{1}{24\pi}\left[\frac{1}{a}\alpha\dot\alpha-\frac{1}{a^2}(\alpha\ddot\alpha+\dot\alpha^2)+\ldots\right]_\tau. \tag{28}$$

One remarks that the expression above can be integrated to yield $\mu$ itself as a purely local quantity in terms of $\alpha$, $\dot\alpha$. Restricting to the leading order, one has the interesting result

$$\mu(\tau)=\frac{\alpha^2(\tau)}{48\pi a}. \tag{29}$$

[5] *ibidem.*

Eq. (28) suggests that $\dot{\mu}$ is zero on uniformly accelerated trajectories $\alpha$ =const. One can write in this case

$$z^+(\tau_1) - z^+(\tau_2) = A \exp B(\tau_1 - \tau_2), \tag{30}$$

$$z^-(\tau_1) - z^-(\tau_2) = \frac{1}{A} \exp \frac{1}{B}(\tau_1 - \tau_2), \tag{31}$$

with $A$, $B$ fixed and of identical sign. One easily verifies that for eqs. (30), (31) the integrand in eq. (23) vanishes. This implies that constant accelerations maintained on intervals significantly larger than $a^{-1}$ lead to a constant mass shift. This is of course not to say that $\mu$ vanishes on these trajectories; it just means that variations of the mass shift are mainly located in the transient phases between pieces of the trajectories with constant acceleration (if the case).

Finally, we mention that the mass shift is a purely relativistic effect: in the non-relativistic approximation of the parantheses in eq. (23), terms linear in velocity turn out to cancel among themselves.

# 5 Discussion

We left unanswered the essential question concerning the sign of $\mu(\tau)$. For sufficiently slowly varying motions, the leading order contribution (29) is always non-negative. Adding the higher orders is not expected to yield a negative quantity, due to the inequalities (27). (Note also that eq. (28) says that if the $a^{-1}$ contribution vanishes, so does the $a^{-2}$ one). One may be inclined to think that positivity of $\mu$ is connected to slowly varying motions. This is not necessarilly so. Consider the trajectory with constant velocities $\beta_i$ for $\tau < 0$ and $\beta_f$ for $\tau > 0$. Then for proper times $0 < a\tau \ll 1$ one finds

$$\mu(\tau) = \frac{a}{4\pi} \left[\gamma_i \gamma_f (1 - \beta_1 \beta_2) - 1\right] (-a\tau \ln a\tau + O(a\tau)) > 0, \tag{32}$$

with $\gamma_{i,f} = (1 - \beta_{i,f}^2)^{-1/2}$. More generally, one can show that the inequality above holds for arbitrary non-uniform $\tau > 0$ trajectories, for $\tau$ sufficiently small.

We haven't succeeded in finding trajectories with $\mu$ assuming negative values. We have neither found a proof to forbid this possibility. In Refs. [1, 8] it was claimed that the mass shift is always positive, irrespective of the mirror model (within the assumption of causality and unitarity). The conclusion was obtained, however, in the context of ignoring renormalization. Note that this implies in eq. (7) the subtraction of the infinite *positive* free field contribution. In this respect the situation is principially the same, e.g., with that of the energy density radiated by a perfectly reflecting moving mirror [11, 12]. The corresponding operator is formally positively defined, but after renormalization negative values are allowed for certain trajectories. If a similar situation happens here, one may contemplate the possibility of generating negative masses, entailing the well-known unphysicalities. (For example, one could imagine the $-m$ negative mass mirror attached to an object of mass $m$ not interacting with the $\phi$ field, both stabilized on an uniformly accelerated trajectory in the absence of external forces; this would represent a system converting the vacuum zero-point energy into the kinetic energy of the accelerated object[6].) We leave open the question of the mass shift sign, hoping to return with further results.

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[6]Theoretically, this is the case for the run-away trajectories of perfectly reflecting mirrors [13, 14]. This is realistically not expected due to the unphysical assumption of perfect reflectivity. When partial reflectivity is taken in to account run-away trajectories are forbidden, see the cited papers.

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